General Education Course Proposal

Proposed Course: Math (Prefix) 45 (No.) What is Mathematics? (Title) **Units** 3

Department: Mathematics **School:** Natural Sciences

GE Category (Indicate one category only):

Foundation: A1___; A2___; A3___; B4 X
Breadth: B1___; B2___; C1___; C2___; D___; E___
Integration: B___; C___; D___; International/Multicultural___

Existing Course___; **Revised Course** X; **New Course**___
Course Included in Current GE Program X

New courses require the Undergraduate Course Proposal form in addition to this form.
Revised courses require the Undergraduate Course Change Request in addition to this form.

Proposed catalog description: Limit course description to 40 words using succinct phrases. Include prerequisites, limitations, lecture/lab hours. Indicate former course number, e.g., (Former Biol 105)

Prerequisite: Students must meet the ELM requirement. Covers topics from the following areas: (I) The Mathematics of Social Choice; (II) Management Science and Optimization; (III) The Mathematics of Growth and Symmetry; and (IV) Statistics and Probability. General Education CORE, Quantitative Reasoning.

Enrollment limit per section: Class room capacity (some large lecture)
Expected number of sections per semester – Year 1 2 ; **Year 3** 3

Attachments:

1. A statement presenting the ways in which this course meets the Specifications provided in the appropriate section of the General Education Policy as well as in the Policies for Inclusion and Evaluation of General Education Courses.
2. A statement of elements common to all sections of this course, identifying content, objectives, required student activities, grading policy, representative texts, and an approximate schedule for the course. Required student activities include such things as papers, research projects, homework, laboratory and/or studio performance, recitations, participation, attendance, and exams.
3. A typical syllabus for a particular offering of the course.
4. Any special cost factors associated with this course.

Approval for Inclusion in General Education

Robert Cromer (signature) — Department Chair — Date 2/20/98
Quamsen (signature) — School Curriculum Committee — Date 2/23/98
(signature) — School Dean — Date 2/26/98
(signature) — General Education Subcommittee — Date 12/15/98
Brandt Kehoe (signature) — Associate Provost — Date 12/22/98

1/14/98

MATH 45: WHAT IS MATHEMATICS?
Spring 1998
Instructor: Dr. Peter Tannenbaum
Office: Peters Business Building 343
Office Hours: Tuesdays: 8:30-9:30 and 3:00-4:00PM;
Thursdays: 8:30-9:30 and 2:00-4:00PM.
e-mail: peter_tannenbaum@csufresno.edu

TEXT

Peter Tannenbaum & Robert Arnold, *Excursions In Modern Mathematics, (Third edition).* Prentice Hall, 1998.

COURSE OBJECTIVES

The purpose of this course is to show the student some of the breadth and scope of modern applied mathematics. This is a survey course, where we will touch upon many ideas. In fact, mathematical ideas and their real-world applications are the primary focus of the course. A second important goal is to give the student a positive experience in the setting of a General Education math course. If you come to class regularly, do all your homework and approach this course with an open and inquisitive mind, you are almost guaranteed to not only pass the course, but to actually enjoy your experience.

COURSE OUTLINE

The material in this course is divided into three independent parts (Parts 1, 2, and 3).
Part 1: Chapters 1, 2, 3, 4 (approx. 5 weeks).
Test 1 (covers Part 1).
Part 2: Chapters 5, 6, 7, 8 (approx. 5 weeks).
Test 2 (covers Part 2).
Part 3: Chapters 9, 10, 11, 12 (approx. 5 weeks).
Test 3 (covers Part 3).

GRADING

Your grade will be based on the following:

- Three tests @ 100 points each.
- Three homework sets @ 20 points each.
- Ten quizzes @ 4 points each.
- TOTAL = 400 points.

A = 340-400 points
B = 300-339 points
C = 260 -299 points
D = 200 -259 points
F: less than 200 points.

WEB PAGE

I will maintain a class web page where you will be able to find course information, your most current grades (posted by the last four digits of your Social Security number), as well as links to other interesting sites for material related to the course. To acces the class web page first go to the Math Department web page (http://www.csufresno.edu/math) and then click on "Faculty and Staff", then on "Peter Tannenbaum", and finally on "Math 45".

About the Homework: Homework is assigned on a regular basis, but you only turn in your homework three times throughout the semester (once for each part of the course). Each homework set is due in class the day of the test for the corresponding part, and is to be done in a blue book (it makes the handling a lot easier). No late homework allowed. The purpose of the homework is to help you assess how well you understand the material and not to punish you for doing it wrong. Thus, if you turn your homework in, and show all your work, you will get full credit for it. Homework is worth a total of 60 points (15% of your grade).

About the Quizzes: There will be ten short (10-15 min.) quizzes given throughout the semester. The quizzes are not graded (you get credit for your work, right or wrong). The purpose of the quizzes is to (a) reward you for being in class, and (b) open up the discussion on a new or important topic. Quizzes are worth a total of 40 points (10% of your grade). There are no makeups for missed quizzes.

About the Tests: Each test will cover one part of the course (4 chapters) and will consist of 25 multiple choice questions. Tests are worth 100 points each. At the end of the semester every student will have an opportunity to make up one of the tests (to bring up their score or to make up a missed test). Other than that, there are no makeups except for compelling reasons (illness or travel on a university sanctioned activity).

COMMON ELEMENTS FOR MATH 45

Math 45 is structured so that approximately 70% of the course contents consists of material that is common to all sections of the course, while the remaining 30% of the course can be chosen at the instructor's discretion.

In practice, the potential course contents come from four independent parts, with each part in turn consisting of four topically related units. *Parts I and II are common to all sections of the course;* instructor's can choose optional topics chosen from Parts III and/or IV. Each of the parts takes approximately 5 weeks to cover, so that the expectation is that every section of the course will cover Parts I and II plus either Part III, or IV, or a combination of these.

Part I. The Mathematics of Social Choice.

This part of the course deals with the applications of mathematical methods to social choice theory. This cluster has important connections with social science, political science, economics and even United States History. The four units discussed in this part of the course are:

1. **The Mathematics of Voting** (approx. 4 hours)

In this unit, students learn about preferential ballots; different voting methods for deciding the outcome of elections with more than two candidates; and different criteria for measuring the fairness of a voting method. The unit concludes with one of the most famous discoveries of social choice theory: *Arrow's Impossibility Theorem.*
In addition, the historical aspects of the subject are explored, with a discussion of the contributions of Condorcet, Borda, Arrow, etc.

The following is a list of the abstract mathematical concepts that are covered in this unit:

- Analysis and representation of data (preference schedules).
- Structure of mathematical problems (input-process-output).
- The nature of a mathematical solution (winner only, partial ranking, total ranking).
- Recursion (recursive ranking methods).
- Non-existence Theorems (Arrow's Impossibility Theorem).

2. **Weighted Voting Systems** (approx. 3 hours).

This unit deals with weighted voting and quantitative methods for measuring the power of individuals, blocs and coalitions. In particular, the computation of the Banzhaf and Shapley-Shubik power distributions is developed in this unit. Applications of these concepts to the Electoral College of the United States, the United Nations Security Council, and local legislative bodies (boards of supervisors, school boards, etc.) are discussed throughout the unit.

The following is a list of the mathematical concepts that are covered in this unit:

- Enumeration of sets and subsets (coalitions, winning coalitions).
- Permutations of a set (sequential coalitions).
- Enumeration of permutations.

- Factorials.

3. **Fair Division** (approx. 3 hours).

This unit deals with mathematical methods for dividing objects among a set of interested parties who have a stake in the division following quantifiable notions of fairness. This is a topic with important applications to social science, economics and political science. The unit covers methods for dividing both continuous and discrete sets of goods. In presenting the material, the historical and geopolitical applications of fair division are considered (this includes the contributions of Hugo Steinhaus, the partition of Europe after WW II, Bosnia, etc.)

The following is a list of the mathematical concepts that are covered in this unit:

- Continuous and Discrete sets.
- Ratio and proportion (the values of shares).
- Areas and volumes of geometric figures.
- Continuous functions (value systems).

4. **The Mathematics of Apportionment** (approx. 4 hours).

The apportionment of seats to states in the House of Representatives is a problem of deep historical and mathematical significance. This unit deals with the mathematical (and historical) issues surrounding the problem of apportionment, and presents some of the classic solutions that have been used in the United States (Hamilton's method, Jefferson's method, Webster's method, and the Huntington-Hill method.) In addition, students learn about the possible flaws and paradoxes presented by the different methods (quota violations, the Alabama paradox, the population, paradox, and the new states paradox). The unit concludes with a discussion of the Balinski-Young Impossibility theorem, which demonstrates the a completely fair apportionment method is a mathematical impossibility. The historical component of the subject is woven throughout the discussion of the mathematics.

The following is a list of the mathematical concepts that are covered in this unit:

- Rounding methods.
- Apportionment Paradoxes (Alabama paradox, population paradox, new states paradox).
- Standard and modified divisors.
- Arithmetic, geometric and harmonic means.

Part II. The Mathematics of Management Science.

This part of the course deals with the applications of mathematical methods to management, allocation and scheduling of resources (usually time, money or space). These topics have important applications to industry, technology, science and engineering. The unifying theme of this chapter is the use of graph and network models to represent problems in which relational information is used. The four units discussed in this part of the course are:

5. **Street Network Problems** (approx. 3 hours)

In this unit, students learn about graph theory and its historical origins (the Konigsberg bridge problem), modeling real world problems with graphs, Euler circuits and Euler paths, Fleury's algorithm, and optimal eulerizations of graphs. The last two concepts are used as a tool to find optimal solutions to certain types of routing problems.

The following is a list of the mathematical concepts that are covered in this unit:

- Representation of relational data (graphs and digraphs).
- Symmetric, asymmetric and totally asymmetric relations.
- Iterative algorithms (Fleury's algorithm).
- Mathematical models (graph models).

6. **The Traveling Salesman Problem** (approx. 3 hours).

One of the most important and famous problems in optimization is the problem of finding optimal Hamilton circuits or Hamilton paths in weighted graphs. This problem is commonly known as the Traveling Salesman Problem, and it has many important and subtle applications in both applied and pure science. In this unit students learn about the nature of the problem, its applications, and several simple algorithms for finding approximate solutions (nearest neighbor, repetitive nearest neighbor and cheapest link algorithms.)

The following is a list of the mathematical concepts that are covered in this unit:

- Approximate and Optimal Algorithms .
- Computational complexity (polynomial vs. exponential complexity) .
- Implementation of algorithms.
- Factorials and their growth.
- Weighted graphs.
- Complete graphs.

7. **Optimal Networks** (approx. 4 hours).

.This unit deals with the problem of connecting a set of locations (cities, terminals, pumping stations, etc.) in the most efficient possible way. The primary applications of this unit are to the design and layout of information, transportation and communication networks. Topics discussed in this unit are trees; spanning trees; minimum spanning trees; Kruskal's algorithm; Shortest Networks; Steiner points, and Steiner trees. The unit involves a combination of ideas and methods from graph theory (trees, spanning trees) and ordinary geometry (angles, Steiner points, Steiner trees).

The following is a list of the mathematical concepts that are covered in this unit:

- Trees.
- Spanning subgraphs and spanning Trees .
- Optimal algorithms (Kruskal's algorithm) .
- Heuristic algorithms.
- Steiner points in triangles.
- Steiner trees and shortest networks.
- The energy minimizing principle.

8. **The Mathematics of Scheduling** (approx. 4 hours).

Scheduling is a pervasive activity in modern life, with obvious applications to practically every area of science and industry.
This unit deals with the basic elements in the mathematical theory of scheduling; the structure of scheduling problems (processors, tasks, processing times, precedence relations), and algorithms for their solution (decreasing time and critical path algorithms).

The following is a list of the mathematical concepts that are covered in this unit:

- Directed Graphs .
- Asymmetric relationships .
- Critical paths.
- Relative percentage of error .
- Incidence matrices .

Part III. Growth and Symmetry (optional topics).

The unifying theme in this cluster is the use of mathematical ideas in the study of natural phenomena: the growth of individual organisms (spiral growth); the growth of entire populations; geometric symmetry, and fractal symmetry.

9. **Spiral Growth in Nature** (approx. 3 hours)

In this unit, students study Fibonacci numbers, the golden ratio, golden rectangles and triangles, gnomons and gnomonic growth. All of these mathematical concepts are tied in with the structure of spiral growing organisms. This topic has important historical connections with Greek and Renaissance art and architecture, which are also covered.

The following is a list of the abstract mathematical concepts that are covered in this unit:

- Fibonacci numbers .
- Explicit description of Fibonacci numbers (Binet's formula) .
- Quadratic equations and the quadratic formula.
- The Golden ratio.
- Fibonacci rectangles and triangles.
- Geometric similarity.
- Gnomons and gnomonic growth.
- Archimedean spirals.

10. **Models of Population Growth** (approx. 4 hours).

This unit deals with mathematical models of population growth: the linear growth model, the exponential growth model and the logistic growth model. In this unit students learn how to work with numerical sequences (both recursively defined and explicitly defined), how to compute partial sums of arithmetic and geometric sequences, and how to analyze the behavior of populations that follow a logistic model of growth.

The following is a list of the mathematical concepts that are covered in this unit:

- Arithmetic sequences (linear growth models).
- Adding the terms of an arithmetic sequence.
- Geometric sequences (exponential growth models).
- Adding the terms of a geometric sequence.
- Explicitly and recursively defined sequences.
- Discrete and continuous models of growth.
- The logistic equation.

11. **Symmetry** (approx. 4 hours).

This unit deals with the notion of geometric symmetry. Understanding the symmetry structure of objects in our world has both a practical and an esthetic component. In this unit students learn about the possible symmetries of two dimensional shapes and how to use the mathematics of group theory to classify shapes and patterns according to their symmetries.

The following is a list of the mathematical concepts that are covered in this unit:

- Rigid motions and transformations of the plane .
- Reflections, rotations, translations, and glide reflections .
- Identity motions .
- Proper and improper motions.
- Composition of transformations.
- Symmetry types.
- Border patterns and their classification.
- Wallpaper patterns and their classification.

12. **Fractal Geometry** (approx. 3 hours).

Fractal shapes are shapes that have the property of self-similarity or symmetry of scale. Many shapes in nature have this property, and fractal geometry is a very new area of mathematics that has important applications in science, medicine and computer graphics. In this unit students learn the construction and properties of such classic fractal shapes as the Koch snowflake, the Sierpinski gasket, and the Mandelbrot set.

The following is a list of the mathematical concepts that are covered in this unit:

- Recursive replacement rules in geometry .
- Self-similarity (symmetry of scale) .
- Non-deterministic geometric constructions (the Chaos Game) .
- Complex Numbers and the complex plane.

Part IV. Statistics.

This part of the course gives a brief presentation of those fundamental notions in statistics that every educated individual is expected to have: understanding some of the complexities involved in the collection of reliable statistical data; how to summarize and describe data; how to compute basic probabilities, and how to interpret and make inferences for normally or approximately normally distributed data.

13. **Collecting Data** (approx. 3 hours)

In this unit, students learn about censuses and surveys; sampling methods (simple random sampling, stratified sampling, systematic sampling); and controlled studies (random, blind, double blind, placebo). The presentation of the material is done through the use of classic case studies (the 1936 Literary Digest Poll; the 1948 Presidential election; the Salk polio vaccine field trials, etc.) which allows the presentation to incorporate the important historical component of the discipline.

The following is a list of the abstract mathematical concepts that are covered in this unit:

- Populations and Samples.
- Parameters and statistics.
- Random sampling.
- Sampling rates.
- Ratio and proportion.

14. **Graphing and Summarizing Data** (approx. 4 hours).

This unit covers some very traditional topics in elementary statistics: graphical descriptions of data (bar graphs, pie charts, stem and leaf plots, box and whisker plots), and numerical summaries of data (mean, median, quartiles, range, interquartile range and standard deviation.)

The following is a list of the mathematical concepts that are covered in this unit:

- Frequencies and relative frequencies.
- Continuous vs. Discrete variable.
- Categorical vs. Numerical variable.
- Measures of location (means, medians, quartiles, percentiles).
- Measures of spread (range, interquartile range, standard deviation).

15. **Basic Probability** (approx. 3 hours).

In this unit, students learn the rudiments of finite probability theory: Sample spaces, events, probability distributions, and finite probability spaces. At the same time, some of the fundamental principles of counting are presented: the multiplication rule, permutations, combinations, and the principle of inclusion-exclusion. This material is highly applied, and the presentation is primarily driven by examples and applications.

The following is a list of the mathematical concepts that are covered in this unit:

- Sets and subsets .
- Unions and intersections of sets.
- Sample spaces and events .
- The multiplication rule .
- Permutations and factorials.
- Combinations and binomial coefficients.
- The binomial theorem.
- Independent events.

16. **The Normal Distribution** (approx. 4 hours).

In this unit students learn the mathematical properties of normal curves and how to use these to study the behavior (including making predictions) for normally distributed data sets. The mathematics in this unit give the foundations for modern sampling techniques, including the reliability of public opinion polls, clinical studies, and casino games.

The following is a list of the mathematical concepts that are covered in this unit:

- Normal curve .
- Standardized data value .
- Sampling distribution .
- Standard error.
- Confidence interval .
- The Central Limit Theorem.