2018-2019 Assessment Report

B.S. in Mathematics

Department and Degree: Department of Mathematics, Bachelor's of Science in Mathematics
Assessment Coordinator: Michael Bishop

1. **Learning Outcomes Assessed this Year**

Direct Assessment
We assessed the following SLOs through embedded questions.
In Fall 2018, we assessed SLOs A1 and A2 by embedded questions on the final exam in MATH 81. We also assessed SLO C2 in MATH 101 by embedded questions on the final exam.

In Spring 2019, we assessed SLOs B2 through embedded questions on the final exam and B3 through presentation rubrics in MATH 111. We also assessed SLO A1 and B2 in MATH 151 through embedded questions on the final exam.

Student Learning Outcomes:

Goal A. Knowledge of Mathematics. Students will gain understanding and conceptual background knowledge of the core areas of mathematics.
Graduates will be able to:

A1. distinguish, describe, and apply the definitions and basic properties of fundamental concepts in algebra, such as set, function, matrix, vector space, group, ring, and field.

A2. distinguish, describe, and apply the definitions and basic properties of fundamental concepts in analysis, such as set, function, continuity, sequence, series, derivative, and integral.

Goal B. Communicating Mathematics. Students will acquire the capacity to read, understand, and write rigorous mathematical proofs and other logical/viable arguments. They will also learn to effectively communicate mathematical ideas orally. Graduates will be able to:

B2. construct mathematical arguments and write clear, organized, and correct mathematical proofs.

B3. discuss and present on a mathematical topic.

Goal C. Applications of Mathematics. Students will learn how to apply their mathematical knowledge to solve theoretical and practical problems. They will learn to develop multiple approaches to difficult problems. Graduates will be able to:

C2. devise and implement strategies to solve practical and/or theoretical problems.

Indirect Assessment

We surveyed and conducted exit interviews of graduating students' perceptions of the program and department.

2. **What assignment or survey did you use to assess the outcomes and what method (criteria or rubric) did you use to evaluate the assignment?**

For direct assessment, we embedded questions into the final exams of MATH 81, 101, 111, and 151 to measure the students' performance on the corresponding SLOs. These questions along with rubrics and results can be found at the end of this document.

For indirect assessment, we conducted exit interviews along with online exit surveys with graduating students to capture student feelings as well as surveys on LaTeX[1] use in MATH 111.

3. What did you learn from your analysis of the data?

MATH 81: Differential Equations with Linear Algebra

- Section 1: 10 out of 27 scored a 70% or higher on A1 question with an average score of 13.3 out of 25 and 14 out of 27 scored a 70% or higher

[1] This is a standard mathematical mark-up language used to write up mathematics articles.

on A2 with an average score of 16.6 out of 25. Overall 19 out of 27 passed the course. Performance on the SLO embedded questions was not statistically significant as a predictive variable in overall course grade.

- Section 2: 16 out of 28 scored a 70% or higher on A1 question with an average score of 16.2 out of 25 and 17 out of 28 scored 70% or higher on A2 with an average score of 17.2 out of 25. Overall 15 out of 28 passed the course. Performance on the SLO embedded questions was statistically significant as a predictive variable in overall course grade.
- Section 3: 14 out of 37 scored a 70% or higher on A1 question with an average score of 5.7 out of 10; 23 out of 37 scored a 70% or higher on A2 part 1 with an average score of 7.2 and 18 out of 37 scored 70% or higher on A2 part 2 with an average score of 5.7 out of 10. Overall 37 out of 37 passed the course. Performance on the SLO embedded questions was statistically significant as a predictive variable in overall course grade.

<u>Summary:</u> Overall, the completion of the SLOs was in line with the completion of the course given that this course has a significant number of non-mathematics majors in them. One instructor noted that the use of complex numbers in the question for SLO A1 seemed to significantly impact students' performance on the question which accounts for the low completion of SLO A1 in Section 1. All instructors noted that students were aware of what was needed to pass the course and likely aimed to do just enough on the final exam to achieve that goal.

MATH 101: Statistical Methods

- Section 1: 23 out of 35 scored 70% or higher on C2. Average of 14.81 points out of 20 for students who took the final exam. Overall, 28 out of 35 passed the class.
- Section 2: 14 out of 34 scored 70% or higher on C2. Average of 12.69 points out of 20 for students who took the final exam. Overall, 26 out of 34 passed the course.

<u>Summary:</u> Performance on the SLO embedded questions was statistically significant as a predictive variable in overall course grade. Overall, the completion of SLO C2 was in line with expectations given that this is an initial course in statistics for mathematics majors.

MATH 111: Transition to Advanced Mathematics

- For SLO B2, 6 out of 21 (29%) scored a 9 or better out of 12 on the embedded question with an average of 6.8 out of 12. A total of 9 out of 23 students passed the class (two did not take the final exam). Performance on the SLO embedded questions was statistically significant as a predictive variable in overall course grade.
- For SLO B3, 16 students were assessed using a presentation rubric at the end of this report. On a scale from Strongly Agree = 5 to Strongly Disagree =1 assigned by the instructor, students averaged as follows:

The student clearly stated the problem	Average of 4.4
The student outlined the solution method clearly	Average of 3.7
The students presented the steps of their solution correctly	Average of 3.7
The students spoke in a clear and confident manner	Average of 3.9
The student answered the question well	Average of 3.3

Summary: Overall, this course is designed to develop the SLO B2 skills. The performance and pass rate are in line with department expectations that this class be difficult in order to appropriately prepare students for more advanced course work. For SLO B3, students performed adequately at the task and the addition of LaTeX to the curriculum appears to serve its intended purpose.

MATH 151 Principles of Algebra

We found the following from the embedded questions on the final exam.

- For SLO A1, 12 out of 32 (38%) scored 14 or higher out of 20 with an average of 11.4 out of 20.
- For SLO B2, 15 out of 32 (47%) scored 14 or higher out of 20 with an average of 10.4 out of 20. Overall 15 out of 32 passed the course.
- Performance on the SLO embedded questions was statistically significant as a predictive variable in overall course grade.

Summary: Overall, the performance on SLOs is lower than expected for students who should be mastering the SLOs. The pass rate is correlated to fulfillment of the SLOs in a statistically significant way which is what would be expected. With the addition of Math 153 earlier in the curriculum, the assessment committee expects this to improve in the future.

Findings from Exit Surveys:
We surveyed graduating majors on their opinions on their completion of the SLOs. We received 22 responses. On a scale from 1 (poor) to 5 (excellent) we asked them whether their educational experience met the following expectations:

- Provide students with conceptual background knowledge in the core areas of mathematics. Students will understand and use the definitions and basic properties of fundamental concepts in algebra and analysis, such as function, derivative, integral, matrix, group. - **Average of 4.7.**
- Teach students to read, understand, and write rigorous mathematical proofs.
    - Students will be familiar with common notations and proof techniques.**-Average of 4.9**
    - Students will read, understand, and be able to reconstruct rigorous proofs of elementary theorems in various areas of mathematics. **-Average of 4.5**
    - Students will be able to write elementary proofs. **-Average of 4.7**
- Provide students with opportunities to apply mathematical knowledge to solve theoretical and practical problems.
    - Students will use their knowledge of calculus and linear algebra to solve practical application problems. **-Average of 4.1**
    - **(**For credential students) Students will use a variety of problem-solving techniques to solve a wide range of problems of both practical and theoretical nature. **-Average of 3.9**
- Develop communication skills, both written and oral, for the purpose of conveying mathematical information.
    - Students will be able to explain their solutions and proofs both orally and in writing. **- Average of 4.2**
    - (For credential students) Encourage a positive attitude towards mathematics teaching and learning. Students will show their excitement and appreciation for the art and science of mathematics. **- - Average of 4.2**

Findings from Exit Interviews:

Students feel well-prepared for their expected career plans. Students who planned on graduate school felt prepared by the difficult course work. Students who planned on becoming teachers enjoyed MATH 149S and CI 161 and felt it was very helpful. They would like more teaching experience before their capstone course in their final semester. They also felt that EHD 50 and MATH 145 were too disconnected given they should be aligned.[2] Students reflected positively on their interactions with classmates and faculty. There was some disdain for the use of videos in flipped classrooms and would have preferred more direct instruction from faculty in those settings.

LaTeX Surveys from MATH 111:

We surveyed students on their experience using LaTeX (a mathematical mark-up language) in Math 111. A total of 39 students surveyed. For the statement, "Prior to this class I have used LaTeX." On a scale of: 4 Frequently, 3 Some, 2 Little, 1 Never, students averaged a response of 1.6, i.e. little to no previous use of LaTeX. For the rest, we use a scale of 5 for Strongly Agree, 4 for Agree, 3 for Neither Agree nor Disagree, 2 for Disagree, and 1 for Strongly Disagree.

I understand what LaTeX is.	Average 4.6
I understand LaTeX basic document classes.	Average 3.7
I understand how to distinguish between a math and a text environment.	Average 4.5
I understand how to create and modify different environments.	Average 3.8
I understand how to incorporate figures and Tables.	Average 2.9
I have or know where to find resources to help me work in LaTeX.	Average 4.6

Students have used LaTeX little to none before MATH 111. They agree and strongly agree that they know what LaTeX is and know how to use it except for weak disagreement with how to incorporate figures and tables. Future

[2] These courses should not be closely aligned according to instructors; the prerequisites are designed to properly sequence courses for students intending to teach mathematics as a career.

instructors may want to include this in the LaTeX aspect of the MATH 111 course.

4. What changes, if any, do you recommend based on the assessment data?

1. Guidance and support for ‘Active Learning’ or ‘Flipped Classroom’ models of instruction.
    a. We commend faculty who implement these models of pedagogy and encourage them to persuade their students of the efficacy of the model.
    b. The assessment committee recommends the department should continue to seek out funding for training and professional development in these instruction models.
    c. The assessment committee suggests that instructors teaching advanced course work (courses depending on MATH 111 as a prerequisite) who wish to use active learning or other innovative pedagogical methods run their courses as a hybrid of these new models and traditional lecture.
2. The assessment committee recommends that MATH 111 instructors also include teaching and assignments to teach the use of figures and tables.

5. If you recommended any changes in your response to Question 4 in last year’s assessment report, what progress have you made in implementing these changes?

1. In response to need to increase usage of technology, the mathematics department has
    a. Created a Mathematical software topics course which began this Fall.
    b. Added a requirement that LaTeX is taught in MATH 111.
2. As part of the SOAP, the department has:
    a. created a rubric for senior projects and final presentations (Measure 2) as part of BS transition. We will pilot it for 2019-2020 and seek department approval next year.
    b. have been in contact with the University Career Development Center for data on Alumni and Employer surveys (Measure 4) rather than conduct our own.
    c. modified Exit survey to include culminating experiences (Measure 6).

6. **Assessment activities to be conducted in 2019-2020 academic year**

1. We will assess MATH 107 for SLO C2 and MATH 171 for SLO A2 in the Fall. We will assess MATH 111 for SLOs B1 and B3 and MATH 152 for SLO A1 and C1 in the Fall.
2. We will adapt the project rubric for MATH 149S for the spring.
3. We will assess all senior projects according to the attached rubric.
4. We will address SOAP measures 8 and 9:
    - Measure 8. We will look at the percentage of students passing MATH 75 or MATH 75A-B, and compare rates between those who took the CRT and those who did the ALEKS program.
    - Measure 9. We will look at passing rates, and whenever possible passing rates in subsequent courses in MATH 111 (which will now have an extra hour of active learning), MATH 6, MATH 75, and MATH 76 all with a recitation section added.

7. What progress have you made on items from your last program review action plan?

A. **Supporting Faculty Research and Workload Issues**
    a. Recommendation: Identify sources for long term funding so the program can offer release time or summer stipends to faculty engaging in research and grant-writing activities.
        i. Last year's response: No progress.
        ii. This year's response: No progress.

B. **Department Budge**t
    a. Recommendation: Identify College and University funds to be included in the departmental funding base for faculty scholarly activities and curriculum coordination.
        i. Last year's response: No progress.
        ii. This year's response: No progress.

C. **Improving technology use in mathematics courses**
    a. Recommendation: Rethink delivery of the calculus, statistics, and upper division courses to include updated use of technology and current mathematical software.
        i. Last year's response: Math 111 requires use of LaTeX. Ad-hoc committee was formed to explore issue.
        ii. This year's response: A Math 191T topics course on programming in mathematics and data science was developed and is currently running in Fall 2019.

D. **Supporting Undergraduate and Graduate Student Research**

a. Recommendation: Rethink the ways to involve undergraduate and graduate into original research rewarding supervising faculty with adequate workload.
    i. Last year's response: No progress.
    ii. This year's response: No progress.

E. Facilities

a. Recommendation: Try to locate all faculty and graduate student offices in closer proximity to the department.
    i. Last year's response: No progress.
    ii. This year's response: No progress.
b. Recommendation: Provide additional space that is equipped appropriately for best practices in teaching mathematics that will facilitate faculty/student collaboration and research activities.
    i. Last year's response: No progress.
    ii. This year's response: No progress.

ATTACH all material here including questions, rubrics, surveys, etc.

Final Exam A1 and A2 Assessment
Math 81, Fall 2018
Instructor: Dr. Doreen De Leon

- SLO A1:

8. (Section 3) (25 points) Use the eigenvalue method to find a real-valued general solution of
$$\mathbf{x}' = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \mathbf{x}.$$

Solution:
$$\begin{aligned} 0 = \det(A - \lambda I) &= \begin{vmatrix} 4-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix} \\ &= (4-\lambda)(2-\lambda) + 1 = \lambda^2 - 6\lambda + 9 = (\lambda - 3)^2. \end{aligned}$$

So, $\lambda = 3$ ($k = 2$).
+2 points for computing $\det(A - \lambda I)$ correctly.
+2 points for factoring and finding the repeated eigenvalue of A.
$\lambda = 3$ Find a basis for the solution space of $(A - 3I)\mathbf{v} = \mathbf{0}$. In matrix form, we have
$$\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$
+1 point for knowing the system to be solved.
Solving:
$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & -1 & 0 \end{array}\right] \xrightarrow{R_2 - R_1 \to R_2} \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right].$$
We see that v_2 is the only free variable, so let $v_2 = r$.
+3 points for identifying v_2 as the free variable and setting $v_2 = r$ or $v_2 =$ some number, like 1.
Then, since $v_1 - v_2 = 0$, we obtain $v_1 = v_2 = r$.
+1 point for evaluating v_1 correctly.
Note that in this case, it is equally correct to identify v_1 as the free variable and then solve for v_2. Both solutions give same credit.
Then, all solutions have the form
$$\mathbf{v} = \begin{bmatrix} r \\ r \end{bmatrix} = r \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
and the eigenvector is
$$\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$
+1 point for correct determination of eigenvector.
One solution is, therefore,
$$\mathbf{x}_1(t) = e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

+2 points for identification of one solution. (Only +1 point if arbitrary constant included.)

We need to find a second, linearly independent solution of the form

$$\mathbf{x}_2(t) = e^{3t}\left(t\begin{bmatrix}1\\1\end{bmatrix} + \mathbf{w}\right),$$

where $\mathbf{w}$ solves

$$(A - 3I)\mathbf{w} = \begin{bmatrix}1\\1\end{bmatrix}.$$

+3 points for knowing what needs to be done to find the second solution.

In matrix form, we have

$$\begin{bmatrix}1 & -1\\1 & -1\end{bmatrix}\begin{bmatrix}w_1\\w_2\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}.$$

+1 point for knowing correct system.

Solving

$$\left[\begin{array}{cc|c}1 & -1 & 1\\1 & -1 & 1\end{array}\right] \xrightarrow{R_2 - R_1 \to R_2} \left[\begin{array}{cc|c}1 & -1 & 1\\0 & 0 & 0\end{array}\right].$$

We have that w_2 is the free variable. If we let $w_2 = 0$, then we have

$$w_1 - w_2 = 1 \implies w_1 = 1.$$

+3 points for identifying w_2 as the free variable and setting $w_2 =$ some number, like 0.

+1 point for evaluating w_1 correctly.

Note that in this case, it is equally correct to identify w_1 as the free variable and then solve for w_2. Both solutions give same credit.

Therefore,

$$\mathbf{w} = \begin{bmatrix}1\\0\end{bmatrix}.$$

+1 point for correct vector.

So,

$$\mathbf{x}_2(t) = e^{3t}\left(t\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}1\\0\end{bmatrix}\right) = e^{3t}\begin{bmatrix}t+1\\t\end{bmatrix}.$$

+2 points for correct determination of $\mathbf{x}_2(t)$.

Finally, the general solution is given by $\mathbf{x}(t) = c_1\mathbf{x}_1(t) + c_2\mathbf{x}_2(t)$, or

$$\mathbf{x}(t) = c_1 e^{3t}\begin{bmatrix}1\\1\end{bmatrix} + c_2 e^{3t}\begin{bmatrix}t+1\\t\end{bmatrix}.$$

+2 points for general solution.

8. (Section 5) (25 points) Use the eigenvalue method to find a real-valued general solution of

$$\mathbf{x}' = \begin{bmatrix}2 & -3\\3 & 2\end{bmatrix}\mathbf{x}.$$

Solution:

$$\begin{aligned} 0 = \det(A - \lambda I) &= \begin{vmatrix} 2-\lambda & -3 \\ 3 & 2-\lambda \end{vmatrix} \\ &= (2-\lambda)(2-\lambda) + 9 \\ &= \lambda^2 - 4\lambda + 13 \\ \implies \lambda &= \frac{4 \pm \sqrt{16 - 4(13)}}{2} \\ &= \frac{4 \pm 6i}{2} \\ &= 2 \pm 3i \end{aligned}$$

+2 points for computing $\det(A - \lambda I)$ correctly.
+2 points for correctly determining the eigenvalues of A.

$\lambda = 2 + 3i$ Find a basis vector for the solution space of $(A - (2+3i)I)\mathbf{v} = \mathbf{0}$.
In matrix form, we have

$$\begin{bmatrix} 2-(2+3i) & -3 \\ 3 & 2-(2+3i) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

or

$$\begin{bmatrix} -3i & -3 \\ 3 & -3i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

+3 points for correctly obtaining the final form of the matrix equation.

Then, we obtain

$$\left[\begin{array}{cc|c} -3i & -3 & 0 \\ 3 & -3i & 0 \end{array}\right] \xrightarrow{R_2 - iR_1 \to R_2} \left[\begin{array}{cc|c} -3i & -3 & 0 \\ 0 & 0 & 0 \end{array}\right].$$

Therefore, we have

$$(-3i)v_1 - 3v_2 = 0 \implies v_1 = -\frac{1}{i}v_2$$

v_2 is a free variable, so let $v_2 = i \implies v_1 = -1$.

+4 points for finding correct values for v_1 and v_2.

So,

$$\mathbf{v} = \begin{bmatrix} -1 \\ i \end{bmatrix}.$$

+1 points for correct determination of eigenvector.

$$\begin{aligned} e^{\lambda t}\mathbf{v} = e^{(2+3i)t} \begin{bmatrix} -1 \\ i \end{bmatrix} &= e^{2t}(\cos(3t) + i\sin(3t)) \begin{bmatrix} -1 \\ i \end{bmatrix} \\ &= e^{2t} \begin{bmatrix} -\cos(3t) - i\sin(3t) \\ -\sin(3t) + i\cos(3t) \end{bmatrix} \\ &= \underbrace{e^{2t} \begin{bmatrix} -\cos(3t) \\ -\sin(3t) \end{bmatrix}}_{\mathbf{x}_1} + i\, \underbrace{e^{2t} \begin{bmatrix} -\sin(3t) \\ \cos(3t) \end{bmatrix}}_{\mathbf{x}_2} \end{aligned}$$

+3 points for knowing to compute $e^{(2+3i)t}\begin{bmatrix}-1\\ i\end{bmatrix}$.
+3 points for knowing that $e^{(2+3i)t} = e^{2t}(\cos(3t) + i\sin(3t))$.
+2 points for computing the product correctly.
+3 points for correctly identifying $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$.
General solution: $\mathbf{x} = c_1\mathbf{x}_1 + c_2\mathbf{x}_2$, or

$$\mathbf{x}(t) = c_1e^{2t}\begin{bmatrix}-\cos(3t)\\ -\sin(3t)\end{bmatrix} + c_2e^{2t}\begin{bmatrix}-\sin(3t)\\ \cos(3t)\end{bmatrix}.$$

+2 points for general solution.

- SLO A2:

9. (Section 3) (25 points) Use the Laplace transform to solve the initial value problem

$$y'' + 4y' + 4y = 28,\ \ y(0) = 2, y'(0) = -3.$$

Solution: Take the Laplace transform of both sides of the equation.

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = \mathcal{L}\{28\}.$$

We have

$$\begin{aligned}\mathcal{L}\{y\} &= Y(s)\\ \mathcal{L}\{y'\} &= sY(s) - y(0) = sY(s) - 2 \quad \textbf{+2 points}\\ \mathcal{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0) = s^2Y(s) - 2s + 3 \quad \textbf{+2 points}\\ \mathcal{L}\{28\} &= \frac{28}{s}. \quad \textbf{+1 point}\end{aligned}$$

If the Laplace transform of 28 is not taken, then 9 points are deducted (since no partial fractions need be done and the inverse Laplace transform is different). If partial fractions are still done, then only -4 points.
Plugging into the Laplace transformed equation gives

$$\begin{aligned}s^2Y(s) - 2s + 3 + 4(sY(s) - 2) + 4Y(s) &= \frac{28}{s}\\ s^2Y(s) - 2s + 3 + 4sY(s) - 8 + 4Y(s) &= \frac{28}{s}\\ (s^2 + 4s + 4)Y(s) - 2s - 5 &= \frac{28}{s}\end{aligned}$$

+3 points for getting this far correctly.

$$\begin{aligned}(s^2 + 4s + 4)Y(s) &= \frac{28}{s} + 2s + 5\\ &= \frac{2s^2 + 5s + 28}{s}\\ Y(s) &= \frac{2s^2 + 5s + 28}{s(s^2 + 4s + 4)}\\ Y(s) &= \frac{2s^2 + 5s + 28}{s(s+2)^2}\end{aligned}$$

+3 points for getting correct expression for $Y(s)$ above.
Then

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{2s^2+5s+28}{s(s+2)^2}\right\} \quad \textbf{+1 point}.$$

We must use partial fractions. We have

$$\frac{2s^2+5s+28}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2}.$$

+3 points for correct form for partial fractions.
Multiply through by $s(s+2)^2$ to obtain

$$\begin{aligned}
2s^2+5s+28 &= A(s+2)^2 + Bs(s+2) + Cs \\
&= A(s^2+4s+4) + Bs^2 + 2Bs + Cs \\
&= As^2 + 4As + 4A + Bs^2 + 2Bs + Cs \\
&= (A+B)s^2 + (4A+2B+C)s + 4A.
\end{aligned}$$

+2 points for getting to this point.
Equate corresponding coefficients:

$$\begin{aligned}
A+B = 2 &\implies B = 2 - A = -5 \\
4A+2B+C = 5 &\implies C = 5 - 4A - 2B = -13 \\
4A = 28 &\implies A = 7
\end{aligned}$$

+3 points for correct calculation of A, B, C.
So,

$$Y(s) = \frac{7}{s} - \frac{5}{s+2} - \frac{13}{(s+2)^2},$$

and

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = 7\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 5\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} - 13\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\}$$

+1 point for knowing the above.

$$y(t) = 7 - 5e^{-2t} - 13te^{-2t}.$$

+2 points for getting 7 and $-5e^{-2t}$ and +2 points for getting $-13te^{-2t}$.

9. (Section 5) (25 points) Use the Laplace transform to solve the initial value problem

$$y'' - 4y' + 4y = 8, \; y(0) = 1, y'(0) = 3.$$

Solution: Take the Laplace transform of both sides of the equation.

$$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = \mathcal{L}\{8\}.$$

We have

$$\begin{aligned}
\mathcal{L}\{y\} &= Y(s)\\
\mathcal{L}\{y'\} &= sY(s) - y(0) = sY(s) - 1 \quad \textbf{+2 points}\\
\mathcal{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0) = s^2Y(s) - s - 3 \quad \textbf{+2 points}\\
\mathcal{L}\{8\} &= \frac{8}{s}. \quad \textbf{+1 point}
\end{aligned}$$

If the Laplace transform of 8 is not taken, then 9 points are deducted (since no partial fractions need be done and the inverse Laplace transform is different). If partial fractions are still done, then only -4 points.

Plugging into the Laplace transformed equation gives

$$\begin{aligned}
s^2Y(s) - s - 3 - 4(sY(s) - 1) + 4Y(s) &= \frac{8}{s}\\
s^2Y(s) - s - 3 - 4sY(s) + 4 + 4Y(s) &= \frac{8}{s}\\
(s^2 - 4s + 4)Y(s) - s + 1 &= \frac{8}{s}
\end{aligned}$$

+3 points for getting this far correctly.

$$\begin{aligned}
(s^2 - 4s + 4)Y(s) &= \frac{8}{s} + s - 1\\
&= \frac{s^2 - s + 8}{s}\\
Y(s) &= \frac{s^2 - s + 8}{s(s^2 - 4s + 4)}\\
Y(s) &= \frac{s^2 - s + 8}{s(s-2)^2}
\end{aligned}$$

+3 points for getting correct expression for $Y(s)$ above.

Then

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{s^2 - s + 8}{s(s-2)^2}\right\} \quad \textbf{+1 point}.$$

We must use partial fractions. We have

$$\frac{s^2 - s + 8}{s(s-2)^2} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{(s-2)^2}.$$

+3 points for correct form for partial fractions.

Multiply through by $s(s-2)^2$ to obtain

$$\begin{aligned}
s^2 - s + 8 &= A(s-2)^2 + Bs(s-2) + Cs\\
&= A(s^2 - 4s + 4) + Bs^2 - 2Bs + Cs\\
&= As^2 - 4As + 4A + Bs^2 - 2Bs + Cs\\
&= (A + B)s^2 + (-4A - 2B + C)s + 4A.
\end{aligned}$$

+2 points for getting to this point.

Equate corresponding coefficients:

$$\begin{aligned} A+B=1 &\implies B=1-A=-1 \\ -4A-2B+C=-1 &\implies C=-1+4A+2B=5 \\ 4A=8 &\implies A=2 \end{aligned}$$

+3 points for correct calculation of A, B, C.

So,

$$Y(s)=\frac{2}{s}-\frac{1}{s-2}+\frac{5}{(s-2)^2},$$

and

$$y(t)=\mathcal{L}^{-1}\left\{Y(s)\right\}=2\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}-\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}+5\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\}$$

+1 point for knowing the above.

$$y(t)=2-e^{2t}+5te^{2t}.$$

+2 points for getting 2 and $-e^{2t}$ and +2 points for getting $5te^{2t}$.

MATH 81 Final Exam Assessment Rubric

1. **SLO A1.** Distinguish, describe, and apply the definitions and basic properties of fundamental concepts in algebra, such as set, function, matrix, vector space, group, ring, and field.

   Problem 10.

2. **SLO A2.** Distinguish, describe, and apply the definitions and basic properties of fundamental concepts in analysis, such as set, function, continuity, sequence, series, derivative, and integral.

   Problems 7, 11.

1. **Problem 10** **10 points** Apply the *Eigenvalue Method* to solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix} \mathbf{x},\ \mathbf{x}(0) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}.$$

   Grade Distribution:

   - **1 points** Set up the *characteristic equation* for finding the eigenvalues of the coefficient matrix.
   - **1 points** Solve the characteristic equation to find one repeated real eigenvalue $\lambda = 3$ of algebraic multiplicity $k = 2$.
   - **2 points** Find one linearly independent *eigenvector* $\mathbf{v}_1$ associated with the *defective eigenvalue* $\lambda = 3$.
   - **2 points** Find a *generalized eigenvector* $\mathbf{v}_2$ associated with the *defective eigenvalue* $\lambda = 3$.
   - **2 points** Build two *fundamental solutions* $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ of the system using the found vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ and write a *general solution* of the system.
   - **2 points** Use the initial conditions to solve the initial value problem.

2. **Problems 7** **10 points** Apply the *Method of Undetermined Coefficients* to solve the initial value problem

$$x'' - x' - 2x = 1,\ x(0) = 1, x'(0) = 0.$$

   Grade Distribution:

   - **1 points** Set up the *characteristic equation*.
   - **1 points** Solve the characteristic equation to obtain two distinct real roots $\lambda_1 = -1$ and $\lambda_1 = 2$.
   - **2 points** Find the *complementary solution* $x_c(t)$ of the nonhomogeneous differential equation, i.e., the *general solution* of the associated (complementary) homogeneous differential equation.

- **2 points** Set up the general form for finding a *particular solution* $x_p(t)$ of the nonhomogeneous differential equation via the *Method of Undetermined Coefficients*.
- **2 points** Find $x_p(t)$.
- **2 points** Write the *general solution* of the nonhomogeneous differential equation and use the initial conditions to solve the initial value problem.

3. **Problems 11** **10 points** Apply the *Laplace Transform Method* to solve the same initial value problem as in **Problem 7**.

   Grade Distribution:

   - **4 points** Apply Laplace transform to both sides of the differential equation and, using its *linearity* and the *Laplace Transform of Derivatives Theorem*, obtain an algebraic equation relative to the Laplace transform $X(s)$ of the unknown function $x(t)$ and solve it for $X(s)$.
   - **3 points** Decompose the right-hand side of the obtained algebraic equation into *partial fractions*.
   - **3 points** Find the solution of the initial value problem via applying inverse Laplace transform and using its *linearity*.

For SLO assessment:

C2. devise and implement strategies to solve practical and/or theoretical problems.

Fall 2018 Math 101 final exam problem #7 (to be assessed):

In its 1994 annual survey of mathematics faculty and students, the American Mathematical Society gave the following data on first-year graduate students who were enrolled in a full-time program offered by a mathematics department in the United States.

	Type of Mathematics Department			
Gender	**Ph.D. (I)**	**Ph.D. (II)**	**Ph.D. (III)**	**Masters**
Male	539	472	715	391
Female	196	249	376	152

The departments were classified according to the highest degree offered (Ph.D. or Masters), and department with a Ph.D. program were further classified according to an assessment rating of the quality of their graduate faculty (I highest). Do the data provide sufficient evidence of a relation between the type of department in which students enroll and their gender? Conduct a Chi-square test of independence at the 0.05 significance level and show all the necessary steps.

Fall 2018 Math 101 Final Exam Problem #7 Grading Rubric

<table>
<tr><td colspan="2">Step 1:
Hypotheses Statement

Score: __/ 2</td><td colspan="2">Null and alternative hypotheses stated correctly

(2)</td><td colspan="2">One of the hypotheses stated correctly and the other one incorrectly
(1)</td><td colspan="2">None of the hypotheses stated correctly

(0)</td></tr>
<tr><td colspan="2">Step 2:
Calculations

Score: ____ /12
(total scores of (a) and (b))</td><td colspan="6"></td></tr>
<tr><td></td><td colspan="7">(a) Calculations of 8 estimated expected cell counts
Score: ____ / 8

Note: Correct calculation of each estimated expected cell counts worth one point.

The number of correct calculations of estimated expected cell counts = the number of points earned (maximum possible points = 8)</td></tr>
<tr><td></td><td>(b) Calculation of the chi-square test statistic

Score:
____ / 4</td><td>Calculation of the chi-square test statistics is correct

(4)</td><td>Formula used is correct, and most parts of calculation are correct
(3)</td><td colspan="2">Formula used is correct, and at least half part of calculation is correct
(2)</td><td>Formula used is correct, and only a small portion of calculation is correct
(1)</td><td>Formula used is incorrect

(0)</td></tr>
<tr><td colspan="2">Step 3:
Rejection region

Score: ____ / 3</td><td colspan="6">Degree of freedom = 1 point
Critical value = 1 point
Statement of rejection region = 1 point</td></tr>
<tr><td colspan="2">Step 4: Decision and Conclusion

Score: ____ / 3</td><td colspan="6">Statement of decision stated correctly = 1 point
or Statement of decision stated incorrectly = 0 point
AND
Statement of conclusion stated correctly = 2 point
or Statement of conclusion sated partially correct = 1 point
or Statement of conclusion stated incorrectly = 0 pint</td></tr>
</table>

Total score for the problem: _______ / 20

Math 111 – Assessment
Spring 2019

There were two section of MATH 111 during Spring 2019. I will refer to them as the 2PM and 3PM Sections.

Section	Students on the roster	Students who took the final	Passed the course
2PM Section	24	21	9 (4 A, 2 B, 3 C)
3PM Section	28	27	13 (1 A, 5 B, 7 C)

In the 2PM Section, there were 5 students repeating the course for the second time. Unfortunately, none of these students passed the course.
In the 3PM Section, there were 11 students repeating the course (one of them for the third time). Only 4 of these students passed the course (3 Cs, 1B).

We assessed one problem for the SLO B2: *Construct mathematical arguments and write clear, organized, and correct mathematical proofs.*

Problem.
A relation R is defined on $\mathbb{Z}$ as follows: Given $a, b \in \mathbb{Z}$, $a\,R\,b$ if and only if $|a-2| = |b-2|$. Prove that R is an equivalence relation on $\mathbb{Z}$ and determine the equivalence classes $[2]$ and $[-3]$.

Scoring Rubric: (12 points total)

- (1 point) Knowing that for a relation to be an equivalence relation, it needs to be reflexive, symmetric, and transitive.
- (2 points) Showing that R is reflexive. Students need to demonstrate that they know that the definition of a reflexive relation involves a quantified statement.
- (2 points) Showing that R is symmetric. To receive full credit, students need to demonstrate that they know they need to prove an implication.
- (2 points) Showing that R is transitive. To receive full credit, students need to demonstrate that they know they need to prove an implication.
- (1 point) Knowing the definition of an equivalence class.
- (1 point) Describing correctly and precisely the equivalence class $[2]$.
- (1 point) Describing correctly and precisely the equivalence class $[-3]$.
- (1 point) Providing an elegant proof for R being an equivalence relation.
- (1 point) Providing thorough, clear and elegant explanations for their answers for the equivalence classes.

Outcomes

Scores for 2PM Section:

- 2, 3, 3, 4, 4, 5, 5, 5, 5, 5, 7, 8, 8, 8, 8, 9, 9, 10, 11, 11, 12
- average score for the class: 6.76 or 56.33%
- Scores of students who passed the course: 5, 8, 8, 9, 9, 10, 11, 11, 12
- Average score for the students who passed the course: 9.2 or 76.6%.

Scores for 3PM Section:

- 2, 2, 2, 3, 4, 4, 4, 4, 5, 6, 7, 7, 8, 8, 9, 9, 9, 9, 9, 10, 10, 10, 10, 10, 11, 11, 12
- average score for the class: 7.22 or 60.16%
- Scores of students who passed the course: 6, 7, 8, 9, 9, 9, 9, 10, 10, 10, 11, 11, 12
- Average score for the students who passed the course: 9.3 or 77.5%.

1. **(SLO A1)**

v. A

(a) Let G be a group, and let H be a subgroup of G. For $a \in G$, give the definition of the *left coset* aH.

(b) Let H be the subgroup $\langle(1\ 3\ 4\ 2)\rangle$ of S_4. Give a complete list of distinct left cosets of H in S_4. Be sure to list the elements in each left coset.

(c) **Prove or Disprove.** The subgroup H in part (0b) is a normal subgroup of S_4.

v. B

(a) Let G be a group, and let H be a subgroup of G. For $a \in G$, give the definition of the *left coset* aH.

(b) Let H be the subgroup $\langle(1\ 3\ 2\ 4)\rangle$ of S_4. Give a complete list of distinct left cosets of H in S_4. Be sure to list the elements in each left coset.

(c) **Prove or Disprove.** The subgroup H in part (0b) is a normal subgroup of S_4.

v. C

(a) Let G be a group, and let H be a subgroup of G. For $a \in G$, give the definition of the *left coset* aH.

(b) Let H be the subgroup $\langle(1\ 4\ 2\ 3)\rangle$ of S_4. Give a complete list of distinct left cosets of H in S_4. Be sure to list the elements in each left coset.

(c) **Prove or Disprove.** The subgroup H in part (0b) is a normal subgroup of S_4.

v. D

(a) Let G be a group, and let H be a subgroup of G. For $a \in G$, give the definition of the *left coset* aH.

(b) Let H be the subgroup $\langle(1\ 2\ 4\ 3)\rangle$ of S_4. Give a complete list of distinct left cosets of H in S_4. Be sure to list the elements in each left coset.

(c) **Prove or Disprove.** The subgroup H in part (0b) is a normal subgroup of S_4.

Rubric: Total 20 points.

Part (a): 7 points

Part (b): 6 points

- Correct elements of H: 1 point
- Correct coset representatives: 2 points
- Correct elements in each coset, including every element of S_4: 3 points

Part (c): 7 points

- Correct conclusion: 1 point
- Correct counterexample: 3 points
- Correct execution of disproof: 4 points

5. **(SLO B2)**

   Let $R = \left\{ \begin{bmatrix} a & b \\ 5b & a \end{bmatrix} : a, b \in \mathbb{Z} \right\}$ and $S = \{a + b\sqrt{5} \mid a, b \in \mathbb{Z}\}$. You may assume without proof that R and S are rings under the usual operations. Prove that R is isomorphic to S.

   <u>Rubric</u>: Total 20 points.

   - Correct definition of function $\varphi \colon R \to S$ (or $S \to R$): 5 points
   - φ is well-defined: 1 point
   - φ is a ring homomorphism: 6 points
   - φ is one-to-one: 4 points
   - φ is onto: 4 points

Project Evaluation Rubric

Student's name: ______________________________

Assessed by: ______________________________ **Date:** ____________________

Attribute	Inadequate (1)	Adequate (2)	Good (3)	Excellent (4)
Quality of Mathematics **Score:** **___/20**	☐ Inadequate literature review, lacking mathematical context	☐ Literature review is present, but mathematical context for current work is not complete	☐ The student has reviewed the literature and explained how current work fits into this context	☐ The student has reviewed the literature and explained how current work fills a gap
	☐ Purpose of the work is unclearly or inadequately explained	☐ Purpose of the work is adequately explained, but may be disorganized	☐ Purpose of the work is clearly explained	☐ Purpose of the work is clearly and thoroughly explained
	☐ Mathematical analysis is inadequate or inadequately explained	☐ Mathematical analysis is adequately explained	☐ Mathematical analysis is mostly comprehensive	☐ Mathematical analysis is thorough and extensive
	☐ No attempt to place results into broader context	☐ Placing of results into broader context is included, but not comprehensive	☐ Placing results into broader context is comprehensively discussed	☐ Placing results into broader context is comprehensively and insightfully discussed
	☐ No attempt to discuss future directions if applicable	☐ Applicable future directions are discussed, but are not comprehensive	☐ Applicable future directions are comprehensively discussed	☐ Applicable future directions are comprehensively and insightfully discussed

Attribute	Inadequate (1)	Adequate (2)	Good (3)	Excellent (4)
Quality of Writing **Score:** **___/20**	☐ Writing is not understandable and engaging to readers ☐ Writing is not organized, inconsistent and does not flow in a logical manner ☐ Paper contains excessive spelling/grammar errors ☐ Organization of paper does not follow standard mathematical format ☐ The writing within each section often belongs in another section	☐ Writing can be clearer and more engaging. ☐ It is possible, but not easy, to follow the main themes of the paper as the writing is mostly logical and consistent ☐ Paper is somewhat free from spelling and grammar errors ☐ Organization of paper somewhat follows standard mathematical format ☐ The writing within each section is generally appropriate for that section	☐ Writing is understandable and engaging to the reader ☐ Writing is organized, consistent, and logical with main themes that are easy to follow. ☐ Paper is virtually free from spelling and grammar errors ☐ Organization of paper mostly follows standard mathematical format ☐ The writing within each section is mostly appropriate for that section	☐ Writing is outstanding, clearly understandable, and engaging to the reader ☐ Writing is extremely well organized, consistent, and logical with main themes that are easy to follow. ☐ Paper is free from spelling and grammar errors ☐ Organization of paper follows standard mathematics format ☐ The writing within each section is appropriate for that section

Attribute	Inadequate (1)	Adequate (2)	Good (3)	Excellent (4)
Quality of Communication to Meet Deadlines **Score:** **____/3**	☐ The communication with project supervisor(s) was poor, and almost all deadlines were not met	☐ Some deadlines were met but the communication with project supervisor(s) was not constant	☐ All deadlines were met and project supervisor(s) was(were) often contacted	☐ All deadlines were met and candidate took initiatives to stay in constant contact with project supervisor(s)

Attribute	Inadequate (1)	Adequate (2)	Good (3)	Excellent (4)
Quality of Presentation **Score:** **____/12**	☐ Slides are unclear, incorrect, or misleading ☐ Oral presentation is incomplete or unclear ☐ Citations are missing or inconsistent	☐ Slides are somewhat clear and appropriate ☐ Oral presentation is somewhat clear and complete ☐ Other's work is referenced consistently, although a few errors exist	☐ Slides are clear and appropriate ☐ Oral presentation is mostly clear and complete ☐ Citations of others' work are mostly consistent and appropriate	☐ Slides are clear and help improve attendees' understanding ☐ Oral presentation is very clear and complete ☐ Citations for others' work are consistent and appropriate
Quality of response to questions **Score:** **____/5**	☐ Responses are incomplete and inaccurate	☐ Responses are accurate but are poorly stated	☐ Responses are accurate and complete and are well stated	☐ Responses are eloquent and skillfully stated
Overall Assessment	☐ Inadequate	☐ Adequate	☐ Good	☐ Excellent

TOTAL: ____/60 **Percentage: ___**

Additional Notes:

Exit Survey for 2018 Graduating Math Majors

Start of Block: Exit survey for graduating math majors

Department of Mathematics
California State University, Fresno

Exit survey for 2018 graduating math majors

All questions are optional, but your responses are greatly appreciated. Positive and negative comments are welcome and suggestions for improvement would be particularly useful. This survey is not anonymous and will take approximately 15 minutes. Please respond prior to your scheduled exit meeting with Dr. Amarasinghe and Dr. Vega. Thank you.

1. Name:

____

2a. Mailing address:

____

____

____

____

____

2b. Fresno State email:

____

2c. Other email:

2d. Cell phone number and home phone number:

3a. Graduation year:

3b. Degree(s) you are graduating with:

3c. Minor(s) if any:

4. Are you enrolled or are you planning to enroll in the Single Subject Credential Program? If so, expected completion year?

5. Are you applying for or considering applying to a graduate program? If so, what program and where? Expected completion year?

6. What are your current post-graduation plans?

7. What are your long-term career goals?

8. Did you work while attending Fresno State? If so, where and in what capacity?

Did you work full-time or part-time?

9a. Did you participate in any of the following activities while at Fresno State?

	Activity (1)
Independent Study in Mathematics (1)	○
Research project in Mathematics (2)	○
REU (3)	○
Math Club (4)	○
Putnam Exam (5)	○
Integration Bee (6)	○
Attended math seminars (7)	○
Presented in math seminars (8)	○
Volunteered at Math Field Day, Department of Mathematics Day or other similar events. (9)	○

9b. Other activities you participated in:

____

____

____

____

____

9c. What other activities would you be interested in?

10. What did you like or dislike about the courses you took at Fresno State? Do you have any suggestions for improvement?

11. Approximately how many times did you meet with your advisor? Was the advising helpful?

12. What advice would you give to new math majors or students considering becoming math majors?

13. On a scale from 1 (poor) to 5 (excellent), please rate below the undergraduate program in terms of how well it helped you achieve the following programs goals and student learning outcomes:

A. Provide students with conceptual background knowledge in the core areas of mathematics.

	Scale from 1 (poor) to 5 (excellent) (1)
Students will understand and use the definitions and basic properties of fundamental concepts in algebra and analysis, such as function, derivative, integral, matrix, group. (1)	

B. Teach students to read, understand, and write rigorous mathematical proofs.

	Scale from 1 (poor) to 5 (excellent) (1)
1. Students will be familiar with common notations and proof techniques. (1)	
2. Students will read, understand, and be able to reconstruct rigorous proofs of elementary theorems in various areas of mathematics. (2)	
3. Students will be able to write elementary proofs. (3)	

C. Provide students with opportunities to apply mathematical knowledge to solve theoretical and practical problems.

	Scale from 1 (poor) to 5 (excellent) (1)
1. Students will use their knowledge of calculus and linear algebra to solve practical application problems. (1)	
2. (For credential students) Students will use a variety of problem-solving techniques to solve a wide range of problems of both practical and theoretical nature. (2)	

D. Develop students' communication skills, both written and oral, for purposes of conveying mathematical information.

	Scale from 1 (poor) to 5 (excellent) (1)
1. Students will be able to explain their solutions and proofs both orally and in writing. (1)	

E. (For credential students) Encourage a positive attitude towards mathematics teaching and learning.

	Scale from 1 (poor) to 5 (excellent) (1)
Students will show their excitement and appreciation for the art and science of mathematics. (1)	

14. Any comments about the above goals and student learning outcomes?

____

____

____

____

____

15. Any other comments or suggestions regarding the mathematics program at Fresno State?

____

____

____

____

____

Q28 **Thank you for completing our survey! We wish you well in your future endeavors!**

End of Block: Exit survey for graduating math majors
