2020-2021 Assessment Report

B.S. in Mathematics

Department and Degree: Department of Mathematics, Bachelor's of Science in Mathematics

Assessment Coordinator: Michael Bishop

1. Learning Outcomes Assessed this Year

Direct Assessment

We assessed the following SLOs through embedded questions. In Fall 2020, we assessed SLO B2 by embedded questions on the exams in MATH 111 and SLO C1 in MATH 81 by embedded questions on the final exam.

In Spring 2021, we assessed SLOs C2 through embedded questions on the exams in MATH 101. We also assessed SLO A1 in MATH 151 through embedded questions on the final exam.

Student Learning Outcomes:

Goal A. Knowledge of Mathematics. Students will gain understanding and conceptual background knowledge of the core areas of mathematics. Graduates will be able to:

A1. distinguish, describe, and apply the definitions and basic properties of fundamental concepts in algebra, such as set, function, matrix, vector space, group, ring, and field.

A2. distinguish, describe, and apply the definitions and basic properties of fundamental concepts in analysis, such as set, function, continuity, sequence, series, derivative, and integral.

Goal B. Communicating Mathematics. Students will acquire the capacity to read, understand, and write rigorous mathematical proofs and other logical/viable arguments. They will also learn to effectively communicate mathematical ideas orally. Graduates will be able to:

B2. construct mathematical arguments and write clear, organized, and correct mathematical proofs.

B3. discuss and present on a mathematical topic.

Goal C. Applications of Mathematics. Students will learn how to apply their mathematical knowledge to solve theoretical and practical problems. They will learn to develop multiple approaches to difficult problems. Graduates will be able to:

C1. apply their knowledge of calculus, differential equations, and/or linear algebra to solve practical problems.

C2. devise and implement strategies to solve practical and/or theoretical problems.

C3. use mathematical software to meet mathematical needs, such as communicating mathematics orally and in writing, performing data analysis, and modeling real life problems.

Indirect Assessment

We surveyed and conducted exit interviews of graduating students' perceptions of the program and department.

2. What assignment or survey did you use to assess the outcomes and what method (criteria or rubric) did you use to evaluate the assignment?

For direct assessment, we embedded questions into the exams of MATH 81, 101, 111, and 151 to measure the students' performance on the corresponding SLOs. These questions along with rubrics and results can be found at the end of this document.

For indirect assessment, we conducted exit interviews along with an online exit survey with graduating students to capture student feelings.

3. What did you learn from your analysis of the data?

MATH 151 Principles of Algebra

We found the following from the embedded questions on the final exam.

 For SLO A1, 17 out of 26 (65%) who took the exam scored 70% or higher on the embedded question with an overall average of 12.69 out of 18 (71%). Performance on the SLO A1 embedded question had an R² of .42 with final exam score and .51 with overall course grade.

<u>Summary:</u> Overall, the performance on SLOs met expectations for students who should be mastering the SLO. The pass rate is correlated to fulfillment of the SLOs in a statistically significant way as expected. Overall, students are mastering SLO A1 as hoped.

MATH 111: Transition to Advanced Mathematics

- Section 1: 11 out of 21 (52%) scored a 70% or higher on the B2 question with an average score of 13 out of 20. Performance on the SLO embedded question had an R² of .56 with final exam score and .49 with overall course grade.
- Section 2: 9 out of 17 (52%) scored a 70% or higher on the B2 question with an average score of 14 out of 20. Performance on the SLO embedded question had an R² of .43 with final exam score and .45 with overall course grade.
- Section 3: 12 out of 24 (50%) scored a 70% or higher on the B2 question with an average score of 13 out of 20. Performance on the SLO embedded question had an R^2 of .25 with final exam score and .21 with overall course grade.

<u>Summary:</u> This course is an intensive proof-writing course that develops the main SLO B2 skills. The performance on the embedded questions and the correlation to course grades are in line with department expectations that this class be difficult in order to appropriately prepare students for more advanced course work.

MATH 81: Differential Equations with Linear Algebra

 Sections 1 & 2 (Same instructor): 22 out of 67 (33%) scored a 70% or higher on the C1 question and with an average score of 15.5 out of 25. Looking more closely at the rubric and student performance, 48 out of 67 (72%) scored 14 points out of 25 which means they would have been able to set-up the diagram and system of equations. Performance on the SLO C1 embedded question had an R² of .55 with final exam score and .18 with overall course grade. Section 3: 25 out of 32 (78%) scored a 75% (instructor-set expectation) or higher on the C1 question with an average score of 9 out of 12.
Performance on the SLO embedded question had an R² of .22 with final exam score and .08 with overall course grade.

<u>Summary</u>: Overall, the completion of the SLO was in line with the final exam grade which meets departmental expectations. Student performance on the embedded questions did vary between the sections, which is likely due to the difference in question structure. Given that Fall 2020 was the first fully virtual semester, the assessment coordinator gave instructors full freedom to structure their exams and questions as they saw fit. In future assessments, the assessment coordinator will work to align the embedded questions more closely.

MATH 101: Statistical Methods

- Section 1: 24 out of 35 (69%) scored 70% or higher on C2 questions on the course quiz. The average score was 10.3 points out of 14. Performance on the SLO C2 embedded questions had an R² of .80 with final exam score.
- Section 2: 32 out of 37 (86%) scored 70% or higher on the C2 question. The average of 16.38 points out of 20 for students who took the final exam. Performance on the SLO C2 embedded question had an R² of .84 with final exam score and .52 with overall course grade.

<u>Summary:</u> Performance on the SLO embedded questions was statistically significant as a predictive variable in overall course grade. Overall, the completion of SLO C2 was in line with expectations given that this is an initial course in statistics for mathematics majors.

Overall Summary of Direct Assessment

Given the unusual circumstances of AY 2020-2021 being fully virtual during a pandemic, student completion of assessed SLO's was in line with department expectations during a normal semester. This points to mathematics students and faculty adapting well to the unusual circumstances and maintaining the same rigorous standards the department has held for decades.

Findings from Exit Surveys:

In an online survey of 9 graduating students:

Statement	Average Score	Responses
Provide students with conceptual background knowledge in the core areas of mathematics Students will understand and use the definitions and basic properties of fundamental concepts in algebra and analysis, such as function, derivative, integral, matrix, group Scale from 1 (poor) to 5 (excellent)	4.56	9
Teach students to read, understand, and write rigorous mathematical proofs 1. Students will be familiar with common notations and proof techniques Scale from 1 (poor) to 5 (excellent)	4.78	9
Teach students to read, understand, and write rigorous mathematical proofs 2. Students will read, understand, and be able to reconstruct rigorous proofs of elementary theorems in various areas of mathematics Scale from 1 (poor) to 5 (excellent)	4.56	9
Teach students to read, understand, and write rigorous mathematical proofs 3. Students will be able to write elementary proofs Scale from 1 (poor) to 5 (excellent)	4.78	9
Provide students with opportunities to apply mathematical knowledge to solve theoretical and practical problems 1. Students will use their knowledge of calculus and linear algebra to solve practical application problems Scale from 1 (poor) to 5 (excellent)	4.56	9
Provide students with opportunities to apply mathematical knowledge to solve theoretical and practical problems 2. (For credential students) Students will use a variety of problem-solving techniques to solve a wide range of problems of both practical and theoretical nature Scale from 1 (poor) to 5 (excellent)	4.67	3
Develop communication skills, both written and oral, for the purpose of conveying mathematical information 1. Students will be able to explain their solutions and proofs both orally and in writing Scale from 1 (poor) to 5 (excellent)	4.67	9
(For credential students) Encourage a positive attitude towards mathematics teaching and learning Students will show their excitement and appreciation for the art and science of mathematics Scale from 1 (poor) to 5 (excellent)	4.75	4

The surveys show that graduates strongly agree that they have learned the skills described in the SLOs.

Findings from Exit Interviews:

Students feel well-prepared for their expected career plans. Students who planned on graduate school felt prepared by the difficult course work. Students who planned on becoming teachers enjoyed MATH 149S and Cl 161 and felt both classes were very helpful. Some students felt the mathematical

programming class should be required for the Applied Math option, potentially replacing the CSCI 41 requirement. There were also requests for a proof-based linear algebra course; Math 152 Linear Algebra mixes proofs and computations.

On the topic of the virtual environment, students had mixed responses. Many missed the engagement of being in-person with classmates and the professor. Many found the asynchronous classes to be less effective due to a lack of interaction. That said, most students said they found virtual office hours to be extremely convenient and were able to attend more often because they didn't need to cross campus or commute. Students also found the virtual recordings and posted notes/whiteboards to be very helpful to rewatch for clarity and correction. Many students also requested that a Discord¹ be set-up for mathematics students to communicate with one another; they also hoped faculty would be able to stop by from time-to-time to provide help.

4. What changes, if any, do you recommend based on the assessment data?

- 1. Faculty should schedule some office hours virtually or find a way to hold office hours simultaneously in-person and virtually.
- 2. Faculty should consider finding ways to record their live classrooms and whiteboards to post recordings online for student reference.

5. If you recommended any changes in your response to Question 4 in last year's assessment report, what progress have you made in implementing these changes?

1. From AY2018-19: Faculty should implement 'Active Learning' - this was tabled due to the transition to entirely online courses in AY 2020-2021.

6. Assessment activities to be conducted in 2021-2022 academic year

- 1. In the Fall semester, we will assess C1 in Math 107, A2 and B1 in Math 171, and A1 in Math 152.
- 2. In the Spring semester, we will assess B2 and C3 in Math 111 and A1 in Math 152.
- 3. We will finalize the project rubric for undergraduate projects.
- 4. We will survey and interview graduating students.
- 5. We will look into a feasible structure for assessment of GE B4 coursework.

¹ Discord is a social media service with text chains and meeting rooms which requires less internet bandwidth.

7. What progress have you made on items from your last program review action plan?

A. Supporting Faculty Research and Workload Issues

- a. <u>Recommendation:</u> Identify sources for long term funding so the program can offer release time or summer stipends to faculty engaging in research and grant-writing activities.
 - i. Last year's response: No progress.
 - ii. This year's response: No progress.

B. Department Budget

- a. <u>Recommendation</u>: Identify College and University funds to be included in the departmental funding base for faculty scholarly activities and curriculum coordination.
 - i. Last year's response: No progress.
 - ii. This year's response: No progress.

C. Improving technology use in mathematics courses

- a. Recommendation: Rethink delivery of the calculus, statistics, and upper division courses to include updated use of technology and current mathematical software.
 - i. Last year's response: All classes were run virtually and used a variety of technology to teach course work.
 - ii. This year's response: Find ways to adapt practices developed in virtual learning into in-person/hybrid models.

D. Supporting Undergraduate and Graduate Student Research

- a. <u>Recommendation:</u> Rethink the ways to involve undergraduate and graduate into original research rewarding supervising faculty with adequate workload.
 - i. Last year's response: No progress.
 - ii. This year's response: No progress.

E. Facilities

- a. Recommendation: Try to locate all faculty and graduate student offices in closer proximity to the department.
 - i. Last year's response: No progress.
 - ii. This year's response: No progress.
- b. Recommendation: Provide additional space that is equipped appropriately for best practices in teaching mathematics that will facilitate faculty/student collaboration and research activities.
 - i. Last year's response: No progress.

ii. This year's response: The department now has eight classrooms that fulfill this recommendation. This progress has been made over the last several years.

ATTACH all material here including questions, rubrics, surveys, etc.

NOTE: Assessments are attached in order which appear in the direct assessment list.

Assessment question for final exam MATH 151 - Spring 2021

(18 points) Consider the direct product of rings $R = \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$ and the set $S = \{(a, b, c) \in R \mid a + c = b\}$.

- (a) Prove that S is a subgroup of R. Is S a normal subgroup of R? Why or why not?
- (b) Prove or disprove that S is a subring of R.
- (c) Show that the mapping $\phi : \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z} \to \mathbb{Z}$ given by $\phi(a, b, c) = a$ is a ring homomorphism. Is ϕ an isomorphism?
- (d) Find the kernel of ϕ . What familiar ring is the factor ring $(\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z})/\text{Ker}\phi$ isomorphic to and why?

Grading Rubric:

- (1 point) Know how elements of the ring R look like.
- (1 point) Know how the addition and multiplication in R are defined.
- (1 + 2 points) Know the Subgroup Test + Use it correctly and rigorously.
- (2 points) Reason that S is normal in R (by using either the definition, or the Normal Subgroup Test, or that R is an Abelian group and that a subgroup of an Abelian group is normal)
- (1 point) Know the definition of a subring.
- (1 point) Disprove that S is a subring by showing that S is not closed under multiplication.
- (2 + 2 points) Show that ϕ preserves addition + multiplication.
- (1 point) Know the definition of an isomorphism and show that ϕ is not injective (either using the definition of an injective function or by showing that the kernel of ϕ is not trivial).
- (1.5 points) Know the definition of the kernel of a homomorphism and find it for the given homomorphism; describe the kernel using set notation.
- (1 point) Use the First Isomorphism Theorem of rings to reason that $(\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z})/\text{Ker}\phi$ is isomorphic to $\phi(\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z})$.
- (1.5 points) Show that ϕ is onto and conclude that $(\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z})/\text{Ker}\phi$ is isomorphic to \mathbb{Z} .

This is a standard question and I am expecting students who are currently at 70% or above in the class and have reviewed for the final to do well in this problem (scoring 70% or above), and missing points only due to lack of being rigorous. Final Exam Math 111 Fall 2020 Section 1 Embedded Quesiton

SLO B2. Students will be able to construct mathematical arguments and write clear, organized, and correct mathematical proofs.

- 3. (20 points). Let R be a relation on \mathbb{Z} defined by xRy if and only if 4|(3x+y).
- a) Prove this is an equivalence relation.
- b) Find the equivalence classes of [1] and [-1].

Rubric: Scoring Rubric: (20 points total)

- (2 points) Knowing that for a relation to be an equivalence relation, it needs to be reflexive, symmetric, and transitive.
- (1 point) Knowing the definition of a reflexive relation.
- (2 points) Showing that R is reflexive. To receive full credit, students need to clearly demonstrate that they are proving a quantified statement.
- (1 point) Knowing the definition of a symmetric relation.
- (2 points) Showing that R is symmetric. To receive full credit, students need to demonstrate that they are proving an implication.
- (1 point) Knowing the definition of a transitive relation.
- (2 points) Showing that R is transitive. To receive full credit, students need to demonstrate that they are proving an implication.
- (2 points) Knowing the definition of an equivalence class.
- (2 points) Describing correctly, clearly, and precisely the equivalence class [1].
- (2 points) Describing correctly, clearly, and precisely the equivalence class [-1].
- (3 points) Using correct and precise terminology and notations throughout the solution.

Math 111 Assessment Section 2

Fall 2020

SLO B2. Students will be able to construct mathematical arguments and write clear, organized, and correct mathematical proofs.

Problem. A relation R is defined on \mathbb{Z} as follows: given $a, b \in \mathbb{Z}$, $a \ R \ b$ if and only if |a - 2| = |b - 2|. Prove that R is an equivalence relation on \mathbb{Z} and determine the equivalence classes [2] and [-3].

Scoring Rubric: (20 points total)

- (2 points) Knowing that for a relation to be an equivalence relation, it needs to be reflexive, symmetric, and transitive.
- (1 point) Knowing the definition of a reflexive relation.
- (2 points) Showing that R is reflexive. To receive full credit, students need to clearly demonstrate that they are proving a quantified statement.
- (1 point) Knowing the definition of a symmetric relation.
- (2 points) Showing that R is symmetric. To receive full credit, students need to demonstrate that they are proving an implication.
- (1 point) Knowing the definition of a transitive relation.
- (2 points) Showing that R is transitive. To receive full credit, students need to demonstrate that they are proving an implication.
- (2 points) Knowing the definition of an equivalence class.
- (2 points) Describing correctly, clearly, and precisely the equivalence class [2].
- (2 points) Describing correctly, clearly, and precisely the equivalence class [-3].
- (3 points) Using correct and precise terminology and notations throughout the solution.

MATH 111, Exam 3. Assessment + Rubric Section 3

Prove that if $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then $3a - 5c^2 \equiv 3b - 5d^2 \pmod{n}$.

Solution. We rewrite what we WTS using the definition of congruence modulo n. We get

$$3a - 5c^2 \equiv 3b - 5d^2 \pmod{n} \iff n \mid (3a - 5c^2) - (3b - 5d^2)$$

Hence, we WTS that

$$(3a - 5c^2) - (3b - 5d^2) = nx$$

for some $x \in \mathbb{Z}$.

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All that is six points
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We know that $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$. We use the definition of congruence modulo n again and get

a-b=ny and c-d=nz (1) for some $y, z \in \mathbb{Z}$. All that is three points

Now we get to work

$$(3a - 5c^{2}) - (3b - 5d^{2}) = (3a - 3b) - (5c^{2} - 5d^{2})$$

= 3(a - b) - 5(c^{2} - d^{2})
= 3(a - b) - 5(c + d)(c - d)
$$\stackrel{(1)}{=} 3ny - 5(c + d)nz$$

= (3y - 5(c + d)z)n

All that is seven points

Hence, as $3y - 5(c+d)z \in \mathbb{Z}$, because it is a sum and product of integers, we get that *n* divides $(3a-5c^2)-(3b-5d^2)$. All that is four points

Math 81 Section 1 Assessment Question and Rubric

Problem 2.(25 pts.)



Our wizard friend Smim and his thieving assistant Bim^1 have had a long, tiring semester learning differential equations at Fresno State. They have decided to celebrate their achievements by mixing up the best fruit punch ever. They are each in charge of their own cauldrons. Smim works with cauldron 1, Bim works with cauldron 2. At time t = 0 cauldron 1 has 200

gallons of water in it, and cauldron 2 has 150 gallons of water. Water flows into Smim's cauldron at the rate of 2 gal/min, and he is adding 1/2 lbs/gal of grapefruit powder to the cauldron while continually mixing. Water also flows into Bim's cauldron, but only at the rate of 1.5 gal/min, but he is adding 3 lbs/gal of grapefruit powder to his cauldron while continuously stirring. Smim's cauldron is leaky, and is losing punch at 3 gal/min. In addition, there is a pipe feeding punch from Smim's cauldron to Bim's cauldron, at 2 gal/min. Bim's cauldron is not leaky, but he is syphoning off the punch (presumably to keep himself in a good mood) at the rate of 1/2 gal/min, and there is also a pipe sending punch from his cauldron back to Smim's, delivering punch at the rate of 3 gal/min. Let $Q_1(t)$ and $Q_2(t)$ be the number of pounds of grapefruit powder in cauldron 1 and 2 respectively.

- (i) Draw a picture of this setup, and label all parts carefully.
- (ii) Write a system of differential equations for Q_1 and Q_2 .
- (iii) Solve the system and describe the limiting behaviors of $Q_1(t)$ and $Q_2(t)$.

Solution: The picture will have 8 processes depicted (various flows and the adding in of the grapefruit powder to each cauldron), for a total of 8 pts. The system of equations the students should arrive at is

$$Q_1'(t) = \left(1 + \frac{Q_2(t)}{50}\right) - \frac{Q_1(t)}{40}$$
$$Q_2'(t) = \left(4.5 + \frac{Q_1(t)}{100}\right) - \frac{7Q_2(t)}{300}$$

¹Art credit: Timothy Peasha

which will earn another 6 pts. Solving the system requires that the students find a fundamental matrix Y (2 pts), invert it (2pts) and compute $\vec{u}' = Y^{-1}\begin{bmatrix} 1\\ 4.5 \end{bmatrix}$ (2 pts). Computing $Y\vec{u}$ (2 pts) gets the solution. They will then have to analyze the long term behavior of the solutions Q_1 and Q_2 for the remaining (3 pts).

Final Exam Problem for the Selected SLO

SLO C1. Students will apply their knowledge of calculus, differential equations, and/or linear algebra to solve practical problems.

Problem 4.

12 points A tank initially contains 40 kg of salt dissolved in 4500 L of water. Brine containing 0.02 kg/L of salt is pumped into the tank at the rate of 50 L/min and the well-mixed solution drains from the tank at the *same rate*.

Set up and solve an initial value problem describing how the amount of salt y (kg) in the tank changes over time t (min).

Rubric:

- **4 points** Set up the initial value problem.
- **6 points** Find a general solution of the corresponding separable differential equation.
- **2 points** Use the initial condition to solve the initial value problem.





(a) $\begin{array}{ll} H_0: \ eta_0 = 0 & (\mbox{The intercept is 0}) \\ H_A: \ eta_0 < 0 & (\mbox{The intercept is less than 0}) \end{array}$



For SLO assessment:

C2. devise and implement strategies to solve practical and/or theoretical problems.

Math 101-05 Final Exam Problem #7, Spring 2021:

7. In its 1994 annual survey of mathematics faculty and students, the American Mathematical Society gave the following data on first-year graduate students who were enrolled in a full-time program offered by a mathematics department in the United States.

Type of Mathematics Department

Gender	Ph.D. (I)	Ph.D. (II)	Ph.D. (III)	Masters		
Male	539	472	715	391		
Female	196	249	376	152		

The departments were classified according to the highest degree offered (Ph.D. or Masters), and department with a Ph.D. program were further classified according to an assessment rating of the quality of their graduate faculty (I: highest). Do the data provide sufficient evidence of a relation between the type of department in which students enroll and their gender? Conduct a Chi-square test of independence at the 0.05 significance level and show all the necessary steps.

Spring 2021 Math 101-05 Final Exam Problem #7 Grading Rubric

Step 1: Hypotheses Statement		Null and alternative hypotheses stated correctly		One of the hypotheses stated correctly and the		tlv	None of the hypotheses stated correctly		
Score:/ 2		(2)		(1)		uy	(0)		
Step 2 Calcu	2: lations			L					
Score (total and (l	:/12 scores of (a) b))								
(a) Calculations of 8 estimated expected cell counts Score:/ 8									
	Note: Correct calculation of each estimated expected cell counts worth one point.								
	The number of correct calculations of estimated expected cell counts = the number of points earned (maximum possible points = 8)								
	(b)	Calculation	Formula		Formula	For	mula	Formula	
	Calculation	of the chi-	used is		used is	use	d is	used is	
	of the chi-	square test	correct, ar	nd	correct, and	cor	rect,	incorrect	
	square test	statistics is	most parts	s	at least half	and	l only a		
	statistic	correct	of		part of	sma	all		
			calculation	n	calculation is	por	tion of		
	Score:		are correc	et	correct	cal	culation		
	/4					is c	orrect		
~ ~		(4)	(3)		(2)		(1)	(0)	
Step 3	3:	Degree of fre	edom = 1 p	oint					
Rejec	tion region	Critical value = 1 point							
G	12	Statement of rejection region = 1 point							
Score	<u> </u>								
Step 4	ep 4: Decision Statement of decision stated correctly = 1 point								
and		or Statement of decision stated incorrectly = 0 point							
Concl	lusion	AND							
		Statement of conclusion stated correctly = 2 point							
Score	Score:/ 3 or Statement of conclusion sated partially correct = 1 point				nt				
	or Statement of conclusion stated incorrectly = 0 pint								

Total score for the problem: ____/ 20